Universal Adiabatic Quantum Computation

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Introduction

Outline

old problems; new solutions

current approach to universal AQC

Types of Gadgets

Improved Gadgets

approaching universal

Universal Adiabatic Quantum Computation

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- Finding exact solutions to certain spin-models plagued researchers
- In 1982 Barahona showed that finding the ground state energy of a spin glass with Ising spins is NP-hard [J. Phys. A: Math. Gen. 15, 3241 (1982)] — considered several topologies and coupling scenarios
- A priori, we might insist on casting this as a milestone in our communities unconventional view of "physical complexity theory"
- Lingua franca uniting physics and computer science elegantly through two sides of the ground state properties of a lattice; computational complexity of determining these properties, and a physical process of doing just that

- Such early observations related to the computational complexity of physical spin systems fostered approaches to solving problem instances through mimicking nature
- Notably, these approaches were based on classical [S. Kirkpatrick et al., Science 220, 671 (1983)] and later quantum annealing [J. Brooke, D. Bitko, T. Rosenbaum, and G. Aeppli, Science 284, 779 (1999)]
- The idea of using the ground state properties of a quantum system for computation found (what appears to be) its full expression in the so called, adiabatic model of quantum computation [E. Farhi, J. Goldstone, S. Gutmann, and M. Sipser, Science 292, 472 (2000)]

is the adiabatic glass half empty, or half full?

- The quantum optimist: There exists a quantum adiabatic process, solving search problem instances quadratically faster than the corresponding classical process [e.g. Roland and Cerf Physical Review A 65 (4), 042308 (2002)]
- The quantum pessimist: There exists a classical computer algorithm simulating such quantum processes with only quadratic overhead [e.g. Sergey Bravyi, David P. DiVincenzo, Roberto I. Oliveira and Barbara M. Terhal, Quant. Inf. Comp. Vol.8, No.5, pp. 0361-0385 (2008)]
- Bipartisanship: How might we use adiabatic quantum processes to solve a problem instance which would be out of reach of a classical computer?

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New ideas relating quantum and classical walks, ground state lattice problems, and computation.

- The autonomous quantum computer proposed by Feynman [Optics News 11, 11 (1982)]
- The problem 5-local Hamiltonian was shown to be QMA-complete by Kitaev [A. Kitaev, A. Shen, and M. Vyalyi, AMS, Graduate Studies in Mathematics 47 (2002)]
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Such methods inspired proof of the polynomial equivalence between the adiabatic and circuit models.

- This modification later inspired a proof of the polynomial equivalence between quantum circuits and adiabatic evolutions by Aharonov et al. [SIAM Journal of Computing, 37(1), 166-194 (2007)] — see also [Mizel, Lidar and Mitchell, Phys. Rev. Lett. 99, 070502 (2007)]
- Kempe, Kitaev and Regev subsequently proved QMA-completeness of 2-LOCAL HAMILTONIAN [SIAM J. Computing 35(5), 1070 (2006)]
- Oliveira and Terhal then showed that universality remains even when the (arbitrary) 2-body Hamiltonians act on particles in a subgraph of the 2D square lattice [Quant. Inf, Comp. 8(10), 0900-0924 (2008)]
- It was next reduced to fairly simplistic interactions (done with Love) [Phys. Rev. A 78, 012352 (2008)] — showed that adding tunable XX interactions renders the D-wave system universal for quantum computation
- Subsequent efforts considered an alternative (dimensionality) reduction of the problem, showing for example QMA-completeness of simulating spin-7/2 particles interacting in a 1D ring [Chase and Landahl, (2008); arXiv:0802.1207]

some challenges

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- The current constructions suffer from
  (i) high irregularity in the required interactions
  (ii) a multitude of additional qubits
- What are the resource estimates (including error correction), required to perform a meaningful computation e.g. one that would beat the best known classical device
- How would this count compare with alternative approaches (such as the gate model)?

- The translation from quantum circuits to adiabatic evolutions began when Kitaev replaced the time dependence of gate model quantum algorithms with spatial degrees of freedom using the non-degenerate ground state of a positive semidefinite Hamiltonian
- The ground state then stores the "history" of the computation

$$|\Psi_{\text{history}}\rangle := |\psi_{\text{in}}\rangle \otimes |0\rangle + U_1|\psi_{\text{in}}\rangle \otimes |1\rangle + \cdots + U_k \cdots U_2 U_1|\psi_{\text{in}}\rangle \otimes |k\rangle \quad (1)$$

- Requires many bits; about a qubit per gate
- Many-body interactions
- Non-physical interactions
- The solution to the many-body interaction and non-physical couplings was found via "gadgets"

Introduction to gadgets

- Basic idea: construct a 2-body Hamiltonian such that its low-lying spectrum is either arbitrarily close or exactly equal to the spectrum of a desired target Hamiltonian
- Gadgets can be used for
  - creation gadgets: creating effective 2-body coupling that cannot be directly realized [Phys. Rev. A 78, 012352 (2008)]
  - reduction gadgets: simulating a k-body Hamiltonian using 2-body Hamiltonians [SIAM J. Computing 35(5), 1070 (2006)]
  - exact gadgets for classical embedding: creating a classical penalty function that embeds a problem instance into quadratic interactions [EPL 99 57004, (2012); Phys. Rev. A 77, 052331 (2008)]

Types of gadgets

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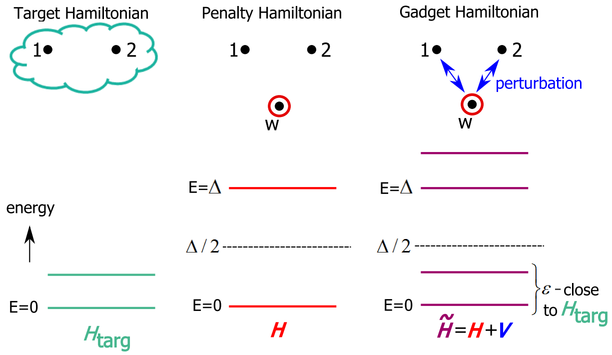
- Perturbative gadgets
  - The method works by constructing a 2-body Hamiltonian such that its leading perturbative contribution gives rise to a desired target Hamiltonian
  - Examples include the subdivision gadget and 3-to-2-local gadget as well as others (see [Jordan and Farhi, Phys. Rev. A 77, 062329 (2008)])
- Non-perturbative gadgets
  - Exactly reduce commuting many-body Hamiltonians to 2-body ones and embed problem instances into Ising spins [EPL 99 57004, (2012); Phys. Rev. A 77, 052331 (2008)]
  - An example is the 3-to-2-local gadget for ZZZ interactions based on ground state spin logic

Subdivision gadget

- Target Hamiltonian: $H_{\text{targ}} = Y + \alpha Z_1 Z_2$.
- Gadget Hamiltonian: $\tilde{H} = H + V$.



Improved subdivision gadget[2]

- Analytical lower bound: $\Delta \geq \left(\frac{2|\alpha|}{\varepsilon}+1\right)(|\alpha|+\varepsilon+2\|Y\|)$.
- Given Y, ε and α, the minimum Δ for yielding the spectral error of ε between $\tilde{H}_-$ and H_{eff} can be numerically found.
- Comparison with Oliveira and Terhal (2008)[1]: $\Delta \geq \frac{(\|Y+|\alpha|\mathbf{1}\|+\sqrt{2}\alpha)^6}{\varepsilon^2}$.

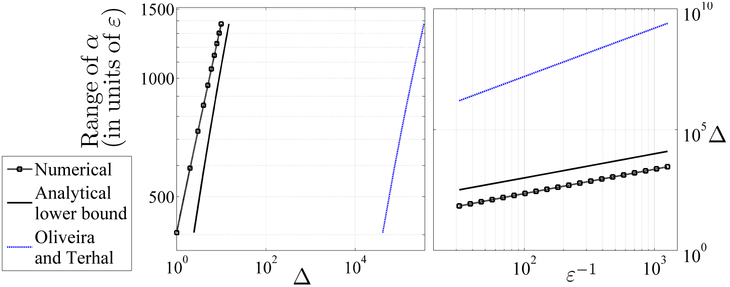


Figure : Left:Range of α as a function of Δ. $\|Y\| = 2$ and $\varepsilon = 0.01$. Right: Δ as a function of ε^{-1}. $\|Y\| = 2$ and $\alpha = 1$.

[1]R. Oliveira and B. Terhal, Quant. Inf, Comp. Vol. 8, No. 10, pp. 0900-0924 (2008).
[2]Y. Cao, J. Biamonte and S. Kais. In preparation.

Improved 3-to-2-local gadget

- Given Y, ε and α, the minimum Δ for yielding the spectral error of ε between $\tilde{H}_{-}$ and H_{eff} can be numerically found.
- In the construction by Kempe, Kitaev and Regev (2006), $\Delta = \varepsilon^{-3}$.

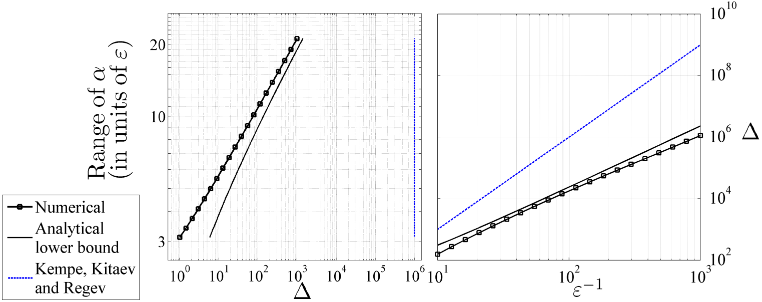


Figure : Left: Range of α as a function of Δ. $\|Y\| = 0.01$ and $\varepsilon = 0.01$. Right: Δ as a function of ε^{-1}. $\|Y\| = 0.01$ and $\alpha = 0.25$.

- An alternative way to simulate $H_{\text{targ}} = Y + \alpha Z_1 Z_2 Z_3$ is to use 5th order perturbative expansion.
- Add an ancilla qubit w. The resulting $\tilde{H} = H + V$ has $H = \Delta|1\rangle\langle 1|_w$ and V taking form of

$$\begin{aligned} V \quad = \quad & Y \otimes \mathbf{1}_w + (J_1 Z_1 + J_2 Z_2 + J_3 Z_3) \otimes |1\rangle\langle 1|_w + \frac{J_4^2}{\Delta}\mathbf{1} \otimes |0\rangle\langle 0|_w \\ - \quad & \frac{J_4^2}{\Delta^2}(J_1 Z_1 + J_2 Z_2 + J_3 Z_3) \otimes |0\rangle\langle 0|_w + J_4 \mathbf{1} \otimes \mathbf{X_w} \\ + \quad & \frac{J_4^2}{\Delta^3}\left[(J_1^2 + J_2^2 + J_3^2)\mathbf{1} + 2(J_1 J_2 Z_1 Z_2 + J_1 J_3 Z_1 Z_3 + J_2 J_3 Z_2 Z_3)\right] \\ - \quad & \frac{J_4^2}{\Delta^4}\left[(J_1^2 + 3J_2^2 + 3J_3^2)J_1 Z_1 + (3J_1^2 + J_2^2 + 3J_3^2)J_2 Z_2 \right. \\ & \quad \left. + (3J_1^2 + 3J_2^2 + J_3^2)J_3 Z_3\right] \otimes |0\rangle\langle 0|_w \end{aligned}$$

- The gadget Hamiltonian can be potentially implemented in experiments since it matches with the form[3] $\sum_i \delta_i X_i + \sum_i h_i Z_i + \sum_{i<j} J_{ij} Z_i Z_j$.
- However, the gadget requires that $\Delta = \varepsilon^{-5}$. For example for $\varepsilon = 0.1$, $\Delta = 10^5$, which is not physical for implementation.

[3] R. Harris et al. Phys. Rev. Lett., 98(177001), 2007

Exact ZZZ gadget

- For simulating $H_{\text{targ}} = -JZ_1Z_2Z_3$, introduce an ancilla qubit w and the exact ZZZ gadget Hamiltonian $\tilde{H} = H + V$ is given as

$$\begin{aligned} H &= \Delta(3\mathbf{1} + Z_1 + Z_2 - 2Z_w + Z_1Z_2 - 2Z_1Z_w - 2Z_2Z_w) \\ V &= J(-\mathbf{1} + Z_1 + Z_2 + Z_3 - Z_1Z_3 - Z_1Z_2 - Z_2Z_3) \\ &+ 2J(\mathbf{1} - Z_w - Z_3 + Z_wZ_3) \end{aligned} \tag{2}$$

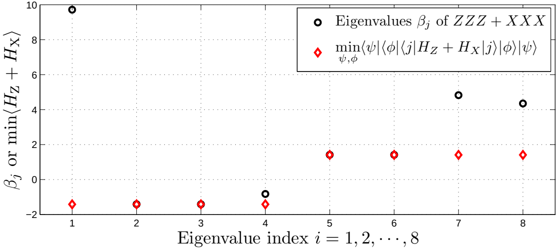
The low-lying spectrum of $\tilde{H}$ exactly captures the spectrum of H_{targ} for $\Delta > 2|J|$.

Applications of exact gadgets

- The gadget Theorem provides a sufficient condition
- If we mix two classical gadgets, each in a different basis (e.g. $\alpha X_1X_2X_3 + \beta Z_1Z_2Z_3$), we must determine if

$$\min_{\psi,\phi}\langle\psi|\langle\phi|\langle j|H_X + H_Z|j\rangle|\phi\rangle|\psi\rangle \neq \beta_j \quad (3)$$

Here $(H_X + H_Z)|j\rangle = \beta_j|j\rangle$, $|\psi\rangle$ and $|\phi\rangle$ are states of the ancilla qubits for H_X and H_Z respectively.



k-body gadgets

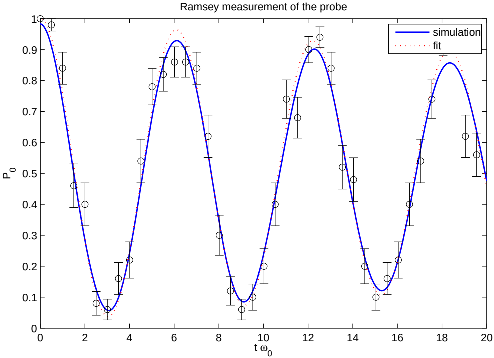
- **Hamiltonian Gadgets with Reduced Resource Requirements**
  with Yudong Cao and others
  Physical Review A 91, 012315 (2015)

The universal adiabatic simulator

- Physics simulation via phase estimation
- Reduce physical requirements

One possible solution

- Adiabatic Quantum Simulators, AIP Advances 1, 022126 (2011)
- Prepares a ground state of a Hamiltonian and then slowing evolves to the same Hamiltonian coupled to a probe (increasing the locality by one)
- The probe is then pulsed with a gate, and read out at different times, reminiscent of Ramsey spectroscopy



[Adiabatic Quantum Simulators, AIP Advances 1, 022126 (2011)].

- What other methods can we think of to avoid the clock construction?
- In other words, adding XX couplers to D-wave hardware has been proven (with Love) to realize a universal quantum computer [Phys. Rev. A 78, 012352 (2008)], but what would we do with it?
- Is there an alternative or extension of method in [Adiabatic Quantum Simulators, AIP Advances 1, 022126 (2011)] which can be realized in near-future D-wave hardware?

- This talk mainly followed:
  Hamiltonian Gadgets with Reduced Resource Requirements
  with Yudong Cao and others
  Physical Review A 91, 012315 (2015)
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- www.QuamPlexity.org